Randomized Methods in Computational Geometry

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Lecture 8: Average conflict size

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Total conflict size

Consider a set of n lines S, a sample R⊆S obtained via Bernoulli trials of parameter p=r/n, and the trapezoidal decomposition H(R).

We have seen an upper bound (n/r)log r to the conflic list size.

We have seen a tail bound to the number of trapezoids with conflict list size more than (n/r)w.

Here we bound the expectation to the sum of the sizes of all the conflist lists.

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Define: $Total(R) = \sum_{\sigma\in H(R)} |L(\sigma)|$

$$E[Total(R)] = E[\sum_{\sigma\in H(R)} |L(\sigma)|] = E\left[\sum_{i=0}^{\infty} \sum_{\substack{\sigma\in H_w(R) \\ w\in[2^i..2^{i+1}]}} |L(\sigma)|\right] \le$$

$$\le \sum_{i=0}^{\infty} 2^{i+1}\left(\frac{n}{r}\right) E[|H_{\ge 2^i}(R)|] \le \left(\frac{n}{r}\right) C \sum_{i=0}^{\infty} 2^{i+1} 2^{-2^i} r^2 \le Cnr \sum_{i=0}^{\infty} 2^{i+1} 2^{-2^i}$$

As in lecture 7 it is easy to see that the summamtion converges to a constant, so the bound is O(nr). On average each trapezoids intersects O(n/r) lines.

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Generalizations

The proof can be generalized to any configuration space of bounded degree and valence.

The proof can be generalized to any c-moment (i.e sums of c-th power of conflict list sizes, for c constant.

Theorem. Given a configuration space Π(S) of bounded degree and valence, a subset R of S obtained via Bernoulli trials of parameter p=n/r, for an integr constant c, holds:

$$E\left[\sum_{\sigma\in\Pi_0(R)} |L(\sigma)|^c\right] \le \left(\frac{n}{r}\right)^c E[|\Pi_0(R)|]$$

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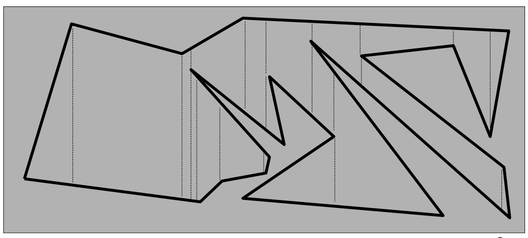
Application: triangulation of simple polygons.

Given a simple polygon with n vertices we produce its trapezoidal map in time O(n log* n).

Where log* n = min {i: $\log^{(i)}$ n < 1}



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- We already saw how to build the trapezoidal decomposition of a set of segments in expected time O(nlog n +k).
- Here k is O(n), thus we have effectively an O(nlog n) method.
- We adopt the variant where we maintain H(S_i) and history(i) to do the point location. All costs except point location are O(n).
- Let $J_i = n/\log^{(i)} n$, J_0=1 and x the least integer for which $J_x > n$, thus x = log* n. Let us consider a phase i for i in [J_k...J_{k+1}]. Suppose that we know in constant time for any point, the trapezoid in H(S_{Jk}) containing it. Then to locate the point in H(N_i) takes time O(1 + log J_{k+1}/J_k) = O($\log^{(k+1)}$ n). The cost for all i in the range is O($J_{k+1} \log^{(k+1)}$ n) = O(n). Since we will pay this cost x times we have a cost O(xn)=O(n log* n).

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Maintaining the invariant.

We have to satisfy the hypothesis. We carry on the usual construction up to level J_k, for each k, at this point we stop and trace N in $H(N_{Jk})$. To do so we locate 1 vertex using history(J_k) in time O(log J_k). Then we trace the polygon sequentially. This step takes time order of the total conflict size.

Since S_{jk} is a random subset of S of size J_k,

we can apply the theorem with $p=J_k/n$. Getting E[Total]< O((np)(1/p))=O(n).

Thus the total cost of the tracing in the group

is O(n). This is repeated log* n times.

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