Lesson 8. Average Total Conflict Size

M. Pellegrini*

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1 Disclaimer

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2 Introduction

In [Mul93] there is a proof of the bound on the expected conflict list size that is almost directly taken from Clakson-Shor [CS89]. A second proof can be obtained via the excess tail bounds in lecture 7, taken form [Mat02] this is shown in the slides.

3 Trapezoidal map of a simple polygon

3.1 A lemma

We need the following lemma (Lemma 3.1.6) for point location in the history of the construction of a trapezoidal map.

Lemma 1 *Let a, b be integers with $1 \leq a \leq b \leq n$. Suppose we have constructed $history(H(S_b)$. For a fixed point q if we know the leaf in $history(H(S_a))$ that contains q, then we can find the leaf containing q in $history(H(S_b)$ in expected time $O(\log \frac{b}{a})$.*

Proof. Call X_j the number of trapezoids in level j that contain q and are generated in phase j of the construction. By backward analysis we know that the expectation of such number is at most $4/j$. The length of the search path from the leaf containing q in $history(H(S_a))$ to the leaf containing q in $history(H(S_b)$ is: $\sum_{j=a}^{b} X_j$. therefore the

*Istituto di Informatica e Telematica del CNR, Via G. Moruzzi 1, 56100-Pisa (Italy). pellegrini@iit.cnr.it

expected value of the length of such path is $O(\sum_{j=a}^{b} 1/j)$, which is $O(\log b - \log a) = O(\log b/a)$.

3.2 Bernoulli trials and random subsets

In order to select a random subset $R \subset S$ of a certain size uniformly at random , we have two main choices. Fix the size r and pick at random $R \in \binom{S}{r}$. Or we can fix the probability $p = r/n$ and take a sample via Bernoulli trial with probability p. In this case the size of the sample is a random variable and $E[|R|] = r$, and the size is close to r with high probability (via a tail bound argument).

Usually analogous result can be found in both models, however the derivation is usually easier in the first model.

An analogous result for the total conflict size can be found in the first model.

References

[CS89] K. L. Clarkson and P. W. Shor. Applications of random sampling in computational geometry, II. *Discrete and Computational Geometry*, 4(1):387–421, 1989.

[Mat02] J. Matoušek. *Lectures on Discrete geometry*, volume 212 of *Graduate Texts in Mathematics*. Springer erlag, New York, 2002.

[Mul93] K. Mulmuley. *Computational Geometry, an Introduction through Randomized Algorithms*. Prentice Hall, 1993.