Lesson 6. Chernoff Bounds and Configuration Spaces

M. Pellegrini*

January 26, 2004

1 Disclaimer

These notes have not been subjected to the usual scrutiny reserved for formal publications. They may be distributed outside this class only with the permission of the Instructor.

2 Introduction

Chernoff bound in Mulmuley [Mul93] Appendix A is applied to:

(i) Sum of $(-1, +1)$ variables with probability $1/2$

(ii) Sum of Bernoulli variables $(0, 1)$ with probability $1/2$. (Binomial distribution)

(iii) Sum of Bernoulli variables $(0, 1)$ with probability p. (Binomial distribution)

(iv) Sum of Geometric variables with probability p. (Negative binomial distribution)

(v) Harmonic distribution. $X_i = 1$ with probability $1/i$, $X_i = 0$ with probability $1 - 1/i$.

(vi) Generalized harmonic distribution (in two special cases). $X_i = \mu(i)$ with probability $1/i$, $X_i = 0$ with probability $1 - 1/i$.

The first special case is when $\mu(i) \leq \bar{\mu}$ is bounded by a constant. In this case $E[X] \leq \bar{\mu} H_n$.

The second special case is when $\mu(i)/i$ is non-decreasing. In this case $E[X] \leq \mu(n)$.

*Istituto di Informatica e Telematica del CNR, Via G. Moruzzi 1, 56100-Pisa (Italy). pellegrini@iit.cnr.it

Chernoff bound in [MR95] are:

(vii) Sum of Bernoulli variables $(0, 1)$ with different probabilities p_i (Poisson trials).

In Hofri [Hof87, page 103-106] one finds Heoffding bounds which apply to the sum of bounded random continuous variables $X = \sum_i X_i$, for $a_i \leq X_i \leq b_i$.

3 Proof of tail estimates for the structural change of incremental algorithms

The proof seems to follow from [CMS92] except that here properties of generating functions are used.

In Mulmuley's book this proof is done from first principles, and at a certain point an intermediate result is very similar to one in the proof for Generalized Harmonic distributions in appendix A.

References

[CMS92] Kenneth L. Clarkson, Kurt Mehlhorn, and Raimund Seidel. Four results on randomized incremental constructions. In *Symposium on Theoretical Aspects of Computer Science*, pages 463–474, 1992.

[Hof87] M. Hofri. *Probabilistic analysis of algorithms.* Springer Verlag, New York, 1987.

[MR95] R. Motwani and P. Raghavan. *Randomized Algorithms.* Cambridge University Press, 1995.

[Mul93] K. Mulmuley. *Computational Geometry, an Introduction through Randomized Algorithms.* Prentice Hall, 1993.