Randomized Methods in Computational Geometry

(Marco Pellegrini)

Lecture 10: Randomized Combinatorial Algorithms for Linear Programming

The problem

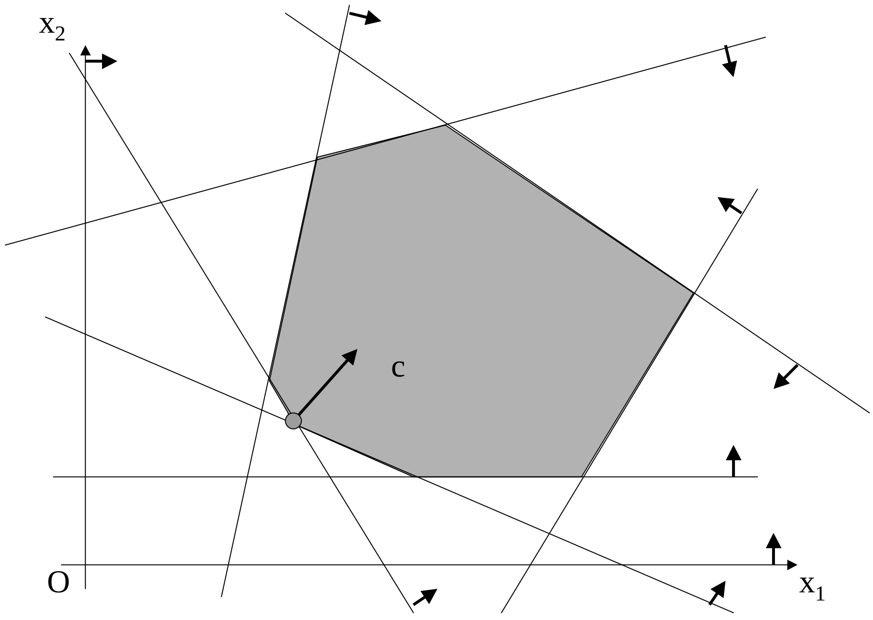
There are many equivalent formulations of the LP problem.

Find $x \in R^d$ minimizing $c \cdot x$

Subject to:

$i = 1,..,d$ $x_i \geq 0$ (positivity constraints)

$i = d+1,..,d+n$ $a_i \cdot x \geq b_i$ (matrix constraints)



Bits of History

- G.B. Dantzig : simplex algorithm, 1940’s.
- Khachiyan : ellipsoid algorithm (1979).
- Karmakar: interior point algorithm (1984).
- Megiddo: LP in linear time (1984).
- Seidel (1990), Clarkson (1995), Matoušek, Sharir and Welzl (1996), Kalai (1992): ramdomized combiantorial algorithms.
- Plus many others...

Clarkson LP_{CI} (JACM 1995)

LP_{CI} (H,B)

{ if (|H|<9d^2) return simplex(H,B);

$r = \lfloor d\sqrt{|H|} \rfloor$; $G = \emptyset$;

repeat

R = randomsubset(H,r);

v = LP_{CI} ($G \cup R$, B);

$V = \{h \in H \mid violates(h,v)\}$

if ($|V| \leq 2\sqrt{|H|}$) $G = G \cup V$;

until ($V = \emptyset$)

return v;}

Observations on LP_{CI}

- It uses an auxiliary algorithm (simplex) as base case , for input size less than 9d^2.
- It needs to ensure that r<|H|.
- It needs to ensure that |G|<|H|.
- The sample size depends on d and |H|.
- The first call is $LP_{CI}(H,H_+)$.

Clarkson LP_{CII} (JACM 1995)

```
LP_CII (H,B)
  { if (|H|<6d^2) then return simplex(H,B);
   r = 6d^2;
   repeat
        R = randomsubset(H,r);
        v = simplex ( R , B);
        V = {h ∈H | violates(h,v)};
        if ( μ(V) < (1/3d) μ(H) )  then
             for all h ∈V, μ(h) =2 μ(h) ;
   until ( V=∅ );
   return v;}
```

Observations on LP_{CII}

- It uses an auxiliary algorithm (simplex) as base case , for input size less than $6d^2$.
- The first call is $LP_{CII}(H,H_+)$.